Tactical Asset Allocation Model for Liquid Alternatives: Seeking Alternative Beta from Alternative Investments in Mutual or Exchange-Traded Funds

Introduction

“Dead are all gods, now we want the superman to live.”

Friedrich Nietzsche, Thus Spoke Zarathustra

Alternative beta (β) is the idea of exposure to risks different from the S&P 500®. “Beta is the elasticity of the portfolio return with the market and presents a linear trade-off between risk and return in the long run.”

This notion was explored in the June 2009 Investment Strategy White Paper, Alternative Investments in Noninstitutional-Sized Accounts.

The Nietzsche quote above could be put in the context of alternative investment in limited partnership (LP) format versus alternative investments in mutual fund/exchange-traded fund (ETF) format: the gods are the LPs and the mutual funds/ETFs are the Übermensch, or superman. Alternatives in mutual fund format were addressed in the May 2009 Investment Strategy White Paper, Goldman Sachs Absolute Return Tracker Index as a Hedge Fund Replicant.

While we recognize the Nietzsche quote as hyperbole, we use it to demonstrate the unmet expectations some investors have with LPs and the grounded realism that traditional investments provide; alternative investments in any format are not supermen, wholly impenetrable to market movements. Market downturns have demonstrated that not all LPs are immune from market turmoil, even though LP managers may advertise them as such. Alpha (α), “the return above the (capital market line) that is in excess of the risk…” is difficult to source, which is one reason LP managers charge a premium (typically, 2% fee/20% carried interest) for it. In contrast, there are many sources of alternative β, many of which are not accompanied by such costs.

In this discussion, we seek to connect the available sources of alternative β on the PNC platform to the philosophy of alternative β in a meaningful, quantitative fashion. To that end, we provide a short numeric example, followed by analytic expressions for the portfolio construction methodology emphasizing the PNC paradigm for alternatives within an investor’s portfolio. We close with a sample performance and a conclusion.

1 Section head quotes are from Friedrich Nietzsche, Thus Spoke Zarathustra.
2 PNC Investment Outlook, You Can’t Always Get What You Want, April 2012.
4 Qian, Hua, and Sorensen, 4.
Alternative β:
A Numerical Example for Diversification

“I teach you the superman. Man is something that is to be overcome.”

β is the magnification of the excess returns of a fund expressed with respect to the market’s excess returns. By definition, β=1 defines the market and β=0 defines something independent of the market. Ideally, alternative β suggests β=0, but this is not practical.

What is sought in this framework is something different than pure market exposure. This is better explained with a simple hypothetical example. Take the S&P 500 and imagine it paired with an investment that provides alternative β. For the purposes of this discussion, we can take the S&P 500 paired with the S&P 500 with a lag. That is, for today’s portfolio, take 50% of the S&P 500 and 50% of the S&P 500 from six months ago. What does this do?

- First, in the example, it preserves the expected return of the portfolio with respect to the market, so the expected return of the portfolio equals the expected return of the pure S&P 500.
- Second, it illustrates the power of diversification without implying an advantage in α. Although the example is a construct, it also isolates the notion of alternative β. Looking at the returns since 1988, the pure portfolio has an average return of 0.85% and a standard deviation of 14.9%. The blended portfolio in our example has the same average return, but it has a standard deviation of 10.1%. The risk, as measured by the standard deviation, fell more than 25%. The β of the lagged S&P 500 is -0.06. Again, the example is illustrative only, but we think it shows the power of alternative β without implying an advantage in α. If we could find something like that to put in a portfolio, its value would be a benefit within a portfolio context.

Where are the sources of alternative β? There are some sources on the PNC platform, such as commodity indexes, specific commodity ETFs, certain long/short managers who are able to package their investments in mutual fund format, and so forth. The challenge is to put them together in a meaningful way and with regard to the overall notion of alternative β. One offering that has become more prevalent is the category of hedge fund replicators. These are specifically constructed to provide alternative β and not necessarily any α. There is no universal way to do this among the different issuers, but most find exposure to global equities, currencies, commodities, and other liquid nonequity instruments.

Portfolio Construction

“...Superman: he is that lightning, he is that frenzy!”

Portfolio construction will usually rely on traditional metrics in a sensible solution space. That is, β measures the returns parallel to the benchmark and α measures the returns perpendicular to the benchmark. This is a geometric interpretation but we believe still a useful one. The manager needs to keep...
the parallel returns less than 1 and make sure the perpendicular returns are reasonably bound.

For the optimization, it is necessary to find something to optimize; this may sound somewhat pedantic, but it is a significant question. Maximizing $\alpha$ would probably lead to a spurious result. Realistically, most investments have some imbedded $\beta$ or market (S&P 500) exposure; it does not make them less useful, in our opinion, but just makes them what they are. Again, the PNC approach is to find alternative $\beta$ in portfolio construction.

$$Constraint: \alpha \in (0, \alpha_{Max})$$

$$Constraint: \beta \in (\beta_{Min}, \beta_{Max})$$

$$Objective \ Function: \text{Minimize: } \varepsilon^T \varepsilon$$

These expressions comprise the constraints and the objective function for the optimization; $\varepsilon$ are the residuals from the regression. There are other details to the framework, but those lines capture the essence of the optimization procedure. The objective function is to minimize the sum of the square of the residuals. Practitioners will likely recognize that as a least squares type of criterion for a good linear fit to a cloud of points. This optimization keeps the metrics meaningful because they define a line. $\alpha$ and $\beta$ exist even on lines for which the fit is poor.

Before looking at the metrics ($\alpha$ and $\beta$) from the fit, it is appropriate that the fit makes sense; that is, the sum of the square of the residuals has been minimized. This may seem obvious, but outliers can wreak havoc with line fits and this procedure manages the outliers and their leverage and influence. In our opinion, it makes little sense to talk about $\alpha$ and $\beta$ without talking about the fit of the lines, whether the fit is good or bad. This framework also keeps the solution in a familiar optimization space, that is, pure quadratic objective function. Optimizations of arbitrary objective functions can have unintended consequences if the sensitivity of the outputs to small changes in the inputs is too high. Because of the way the problem was phrased, the sensitivity of the solution to small perturbations in the inputs is understood and minimized because it is a pure quadratic objective function. It is less clear what happens if another objective function is used (for example, maximizing F-value, chi-squared number).

How are $\alpha$ and $\beta$ computed? This is done through ordinary least squares (OLS). Taking $R_M$ as the returns of the market (plus a dummy column of 1s to account for $\alpha$), the Alternative (ALT) TAP returns as $R_{TAP} = R_{ALT} w$, where $R_{ALT}$ is the matrix of returns for the 40 Act funds and ETFs and $w$ is a weights vector. The simple equation with which to begin is this:

$$R_{TAP} = R_M \beta$$

---


After the appropriate substitution and rearrangement, the equation is:

$$(R_M^T R_M)^{-1}R_M^T R_{ALT} w = \beta$$

$\beta$ represents both $\alpha$ and $\beta$ in vector format. This reveals that $\alpha$ and $\beta$ are linear functions of the weights. The residuals, $\varepsilon$, follow naturally from this and simply are:

$$\varepsilon = (R_{ALT} - R_M (R_M^T R_M)^{-1}R_M^T R_{ALT})w$$

Again, this is a linear function with respect to $w$. $\varepsilon^T \varepsilon$ is an elementary exercise, which demonstrates that the sum of squared residuals is quadratic with respect to $w$, hence the quadratic objective function. The solutions for $\alpha$, $\beta$, and $\varepsilon$ span the space of all possible lines. If $\varepsilon^T \varepsilon$ were maximized instead, this would still yield a valid ordinary least squares fit, but would probably be the solution filled with large outliers of large influence and leverage. This is why minimizing $\varepsilon^T \varepsilon$ was chosen instead. These expressions provide the quadratic objective function and linear constraints needed for optimization.

The details of $\alpha_{\text{Max}}$, $\beta_{\text{Min}}$, and $\beta_{\text{Max}}$ are matters of judgment. What is important in this case, we believe, is that the solution space is being defined precisely in terms of $\alpha$ and $\beta$ in a way that preserves the quality of the fit for them. This also could have been done, for example, with a t-stat condition, F type constraint, chi-squared considerations, or Jarque-Bera metrics, but those would generate highly nonlinear (degree $> 2$) and transcendental functions into the problem, which we believe would be a complication. While the mathematics is unremarkable, the hope is that there is an appreciation for the care with which the solution space was defined and searched.

Using this methodology helps to define weights for the purposes of portfolio construction. These weights are seen in Chart 1 (page 5). Here are the various alternative investments and the evolution of the respective weights over time. Because of the short history of some of these investments, there are limitations to the length of the back test. For example, rebalancing occurs every 13 weeks and an in-sample period of 52 weeks is used to calibrate the new weights for the subsequent 13 weeks. An interesting result, in our view, is that the hedge fund replicators and multistrategy funds seem to form a type of sector exposure as does the long/short space (a replicator “complex” and a long/short “complex”). Allocating to a sector seems to ensure that the sum of the weights within that sector is approximately constant. For instance, the replicators/multistrategy’s sector may be such that all the allocation is shared between the two, that is, $w_{GS \text{Replicator}} + w_{AQR} = K_1$. For the long/short sector, we have $w_{Diamond Hill Long/Short} + w_{JPM\text{-EquityMarketNeutral}} = K_2$. This may be an accident of the back test or a result of the construction process for the replicators used by the providers. Whatever the reason, it seems to hold true in our back test. Again, it should be noted that there are no constraints on the individual weights other than nonnegativity and full investment. The sector phenomenon fell out of the returns information and nothing else. If this result is robust to further updates in the model, it will be even more curious. The model seems to be controlling for $\beta$ by seeking out the long/short and equity market neutral funds. Additional $\beta$ and/or $\alpha$ are picked up by the replicator and multistrategy funds.
Chart 1
Weights of Available Alternative Investments in Mutual Fund or ETF Format

- Barclays Capital U.S. Aggregate Index (LBUSTRUU)
- Van Eck Global Hard Assets Fund (GHAIX)
- PowerShares DB Energy Fund (DBE)
- PowerShares DB Agriculture Fund (DBA)
- SPDR Gold Trust (GLD)
- iPath® DJ-UBS Commodity Index Total ReturnSM ETN (DJP)
- iShares Barclays TIPS Bond Fund (TIP)
- Vanguard Inflation-Protected Securities Fund (VIPSX)
- PIMCO Commodity Real Return Strategy (PCRIX)
- DWS RREEF Global Estate Securities Fund (RRGTX)
- ING Real Estate Fund (CRARX)
- Neuberger Berman Real Estate Fund (NBRIX)
- T. Rowe Price Real Estate Fund (TRREX)
- JPMorgan Global Research Market Neutral Fund - Institutional (JPMNX)
- Diamond Hill Long-Short Fund (DHLSX)
- Turner Spectrum Fund (TSPEX)
- Natixis ASG Global Alternatives Fund (GAFYX)
- Dreihaus Active Income Fund (LCMAX)
- Eaton Vance Global Macro Absolute Return Fund (EIGMX)
- AQR Managed Futures Strategy Fund (AQMX)
- AQR Multi-Strategy Alternative Fund (ASAIX)
- Goldman Sachs Absolute Return Tracker Fund (GIRTX)

Source: Bloomberg L.P., PNC
How does our theoretical approach fare against reality? We can run the weights out of sample and see whether the $\beta$ and $\alpha$ constraints hold out of sample. This is done in Chart 2.

Chart 2 demonstrates that a reasonable line is achieved. The $\alpha$ and $\beta$ are 0.002 and 0.48 with accompanying t-values of 0.64 and 36.1 (R-squared = 0.94). The work that was put into minimizing the sum of the square of the residuals seems to have paid off in the form of a good fit. The $\beta$ constraint is satisfied out of sample. The $\alpha$ constraint is less clear since the t-value is below 2; it looks indistinguishable from 0 at a certain confidence level.

Another reason to ground the expectations around $\alpha$ generation concerns these types of alternatives in mutual fund/ETF form; they are designed by the manufacturers for alternative $\beta$. If the aim was to build a portfolio around alternative $\beta$, then it appears that the goal was achieved. The uncertainty in $\alpha$ being different from zero is a by-product of the priority list in the order of portfolio construction’s importance (definitely $\varepsilon_i$ first, then $\beta$, and then maybe $\alpha$).

Apart from the $\beta$ for the entire out-of-sample period, some insight could be gained for the rolling 52-week $\beta$, that is, rolling 52-week periods within the out-of-sample period. These are depicted in Chart 3. The scale on the left from 0.45 to 0.55 mimics the $\beta$ constraint in the optimization. As is evident from the graph, the $\beta$ stays within the window for most of that period. For the most part, the procedure constrains $\beta$ as expected. Variations in $\beta$ might be due to a particularly volatile market period or abnormalities in the constituents, which are themselves portfolios of other securities. This graph gives some credence toward the philosophy of portfolio construction for the ALT TAP model in targeting alternative $\beta$.

The choice of 0.45 and 0.55 as the lower and upper bounds for $\beta$ deserves some remark. A portfolio of 65% S&P 500 and 35% Barclays Aggregate Bond indexes produces a $\beta$ of about 0.5. The HFRI Fund of Funds Composite Index has a $\beta$ of 0.25. In our opinion, the reality of many of these new mutual fund/ETF alternatives is that the benchmark is not clear. All other things being equal, our alternatives portfolio should probably have a $\beta$ less than that of the 65/35 portfolio. Realistically, we believe reaching the HFRI’s beta of 0.25 is probably unattainable. The compromise between 0 and 1 is about 0.5. As said before, this is a matter of judgment. To be more specific would probably be to engage in false precision; to be less specific would probably be to deviate from goodness.

---

7 The overall weight of the alternatives allocation in the PNC Asset Management Group Balanced Portfolio is a proportional weighting of stocks and bonds in about a 65-35 ratio.
Alpha and Beta

"I guess that you will call my superman a devil!"

Our $\alpha$ and $\beta$ are quantities from an OLS regression and nothing else. Other types of $\alpha$ and $\beta$ are computed through a variety of means, including Bayesian, robust, generalized least squares, and Kalman filters. Analytically, these quantities do not always fit within a generic optimization engine without some effort because of the high degree of nonlinearity. We see nothing wrong per se in using them as such. However, simplicity has its advantages, and we believe that most people are not acquainted with these forms of $\alpha$ and $\beta$. Having an intuition about the answer is almost as important as the answer itself. That is a stylistic choice but one that we think should carry the portfolio through the challenging periods.

The ability to find a good fit determines how useful $\beta$ is, and the usefulness of $\beta$ determines the usefulness of $\alpha$.\(^8\) That is, one’s ability to control the $\varepsilon_i$ affects one’s ability to control $\beta$, which in turn affects one’s ability to control $\alpha$. In the end, $\alpha$ and $\beta$ mean much less if the $\varepsilon_i$ are not reasonable; hence, the above method putting $\varepsilon_i$ in the objective function rather than any condition on $\alpha$ or $\beta$. To the practitioner, this might seem intuitively obvious, but an explicit calculation might make the point transparent to the generic reader. Without any fanfare for the obvious and known, the traditional OLS solution for a set of points, $(x, y)$, is presented as:

$$a = \frac{1}{\Delta} \left[ \frac{\Sigma y_i}{\Sigma x_i y_i} \right] = \frac{1}{\Delta} (\Sigma x_i^2 y_i - \Sigma x_i \Sigma y_i)$$

$$b = \frac{1}{\Delta} \left[ \frac{N \Sigma y_i}{\Sigma x_i y_i} \right] = \frac{1}{\Delta} (N \Sigma x_i y_i - \Sigma x_i \Sigma y_i)$$

$$\Delta = \left| \frac{N \Sigma x_i^2}{\Sigma x_i^2} \right| = N \Sigma x_i^2 - (\Sigma x_i)^2$$

Ignoring systematic errors which would introduce correlations between the uncertainties, the standard deviation $\sigma_i$ of the determination of any parameter $z$ is given as the root sum square of the products of the standard deviation of each data point $\sigma_i$ multiplied by the effect which that data point has on the determination of the parameter $z$.\(^9\)

$$\sigma_z^2 = \sum_i \left[ \sigma_i^2 \left( \frac{\partial z}{\partial y_i} \right)^2 \right]$$

Noting that $\sigma_i = \sigma$, or $\sigma^2 \approx \frac{1}{N-2} \Sigma (y_i - a - b x_i)^2$ and computing the partial derivatives, we have:


\(^9\) Bevington, 9.
\[
\frac{\partial a}{\partial y_j} = \frac{1}{\Delta} \left( \Sigma x_i^2 - x_j \Sigma x_i \right)
\]

\[
\frac{\partial b}{\partial y_j} = \frac{1}{\Delta} \left( N x_j - \Sigma x_i \right)
\]

With the appropriate substitutions, we can see for \( \sigma_a \) and \( \sigma_b \) that the appropriate first order approximations are:

\[
\sigma_a^2 \approx \sum_{j=1}^{N} \frac{\sigma^2}{\Delta^2} \left[ (\Sigma x_i^2)^2 - 2x_j \Sigma x_i^2 \Sigma x_i + x_j^2 (\Sigma x_i)^2 \right]
\]

\[
= \frac{\sigma^2}{\Delta^2} \left[ N(\Sigma x_i^2)^2 - 2(\Sigma x_i)^2 \Sigma x_i^2 + \Sigma x_i^2 (\Sigma x_i)^2 \right]
\]

\[
= \frac{\sigma^2}{\Delta^2} (\Sigma x_i^2) \left[ N \Sigma x_i^2 - (\Sigma x_i)^2 \right] = \frac{\sigma^2}{\Delta} \Sigma x_i^2
\]

\[
\sigma_b^2 \approx \sum_{j=1}^{N} \frac{\sigma^2}{\Delta^2} \left[ N^2 x_j^2 - 2 N x_j \Sigma x_i + (\Sigma x_i)^2 \right]
\]

\[
= \frac{\sigma^2}{\Delta^2} \left[ N^2 \Sigma x_i^2 - 2 N (\Sigma x_i)^2 + N (\Sigma x_i)^2 \right]
\]

\[
= \frac{N \sigma^2}{\Delta^2} \left[ N \Sigma x_i^2 - (\Sigma x_i)^2 \right] = N \frac{\sigma^2}{\Delta}
\]

After rearrangement, it is clear that the uncertainty in the line itself, \( \sigma \), filters to the uncertainty in \( \beta \) (\( \sigma_\beta \sim N \frac{\sigma^2}{\Delta} \)), which filters to the uncertainty in \( \alpha \) (\( \sigma_\alpha \sim N \frac{\sigma^2}{\Delta} \Sigma x_i^2 ), an intuitively obvious result made explicit with a back-of-the-envelope calculation. This type of clarity may be lost on more exotic forms of \( \alpha \) and \( \beta \), which is another reason we wish to stay within the OLS framework. The equations lead to putting \( \sigma \) in the objective function of the optimization and placing \( \alpha \) and \( \beta \) in the constraints of the optimization. The result also tempers expectations around \( \alpha \). \( \alpha \) is the ultimate goal to be sure, but it is dependent on many other quantities in this framework, which is why PNC seeks a portfolio attuned to alternative \( \beta \), not \( \alpha \). Someone selling an investor \( \alpha \) should put equal effort into selling an investor equivalent if not better confidence in their ability to deliver \( \beta \). Again, we are looking for consistent \( \beta \) and persistent \( \alpha \), in an ideal world.\(^\text{10}\)

\(^{10}\) At this point, we would be remiss if we were to omit nonperformance-related sources of alpha. For a complete discussion of these, please refer to the white paper by Investment Advisor Research entitled Selecting the Managers: Research and Due Diligence Process.
The aim of this piece was to present a self-consistent portfolio construction methodology in concert with the philosophy of alternative $\beta$ at PNC. Such a portfolio could be made by using alternative investments in mutual fund or ETF format.

Although $\alpha$-capture from these types of vehicles is not guaranteed and perhaps even ought not to be expected, we believe they provide exposure to nontraditional, liquid sources of return. These alternative sources of return can likely reduce risk in portfolios with the potential to add to returns.

Investors should also keep in mind their future goals, income needs, and risk tolerance. Alternatives in mutual fund/ETF form are relatively new, but we believe their potential risk mitigating characteristics make them attractive as an addition to portfolios for investors ready to embrace alternative risks.

11In addition to the other cited works, interested readers may find additional information on the topic discussed in Gene H. Golub and Charles F. Van Loan, *Matrix Computations.* (Baltimore: The John Hopkins University Press, 1996).

12 Definitely not said by Nietzsche.